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A model to predict crop response to applied fertilizer nutrients in heterogeneous fields

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Abstract

In this paper we develop a model to quantify spatial variability in indigenous soil nutrient supply and assess the impact of this heterogeneity on fertilizer use efficiency with uniform or site-specific nutrient application. Utilizing field data for wheat and rice response to applied N and cotton response to applied K, the model predicts that the magnitude of the difference in the nutrient input requirement of a heterogeneous field for site-specific versus uniform nutrient application depends on (1) a curvilinear crop response to nutrient supply and the mathematical form of the response function, (2) the degree and spatial distribution of the nonuniformity in native soil-nutrient supply as quantified by its variance and skewness, (3) the targeted yield level, and (4) the effectiveness of fertilizer-nutrient addition, quantified by the slope of the relationship between the net increase in actual nutrient supply available to the crop and the quantity of applied nutrient.

Introduction

Commercial production of agronomic crops in the USA is mainly practiced on an extensive scale. Single fields that are managed as a unit often exceed 50 ha. Within such large fields, the nutrient supplying capacity of soil (hereafter called the indigenous nutrient supply) may vary markedly as a result of landscape position and erosion in rainfed systems, and soil leveling operations on irrigated land. Despite such heterogeneity, present soil fertility management relies on uniform application of fertilizer-nutrient inputs to correct crop nutrient deficiencies.

Variable application rates to match the nutrient input requirements of different locations within a nonuniform field have been proposed to improve fertilizer use efficiency [13]. Development of field equipment to achieve this capability is in progress. There is a need, however, for a theoretical framework to assess the economic benefit from site-specific nutrient application to a heterogeneous field.

Although there is a considerable body of literature on statistical methods for the measurement and description of spatial variability [1, 7, 9, 13], the effect of soil heterogeneity on crop yields, output/input efficiency, and economic return has received less attention. In studies that evaluated these performance parameters, heterogeneity was found to decrease economic return [6] and biological efficiency [14, 16] from applied inputs compared to returns and efficiency in a homogeneous field.

The object of this paper is to present a theoretical basis for evaluating yield and fertilizer-nutrient utilization efficiency (FUE, defined as the ratio of Δyield/Δnutrient input) in relation to the degree of spatial variability in the indigenous nutrient supply. We define the indigenous supply as the quantity of nutrient that the crop derives from soil, although this quantity may change with fertilizer addition. The model we develop couples a probabilistic characterization of spatial variability in soil nutrient availability with a crop yield response function to indigenous and applied nutrient supply so that yield and FUE from uniform versus site-specific nutrient application methods can be compared. Several case studies are used to illustrate the model output, and to identify the dominant factors that influence crop response to applied nutrient in a heterogeneous field.

Materials and methods

Development of the model

Let x be the coordinate vector of a point in a field, and let $z(x)$ be the indigenous nutrient supply of a particular nutrient at that point. Soil nutrient status may vary with position due to both deterministic and random influences [10]. Whatever the source of variation, however, statistics associated with indigenous nutrient supply, such as its mean, variance, and coefficient of skewness, may be computed as if $z(x)$ were a random variable. Our approach in this paper is to avoid explicit representation of z on x, and instead focus on the frequency distribution of indigenous soil nutrient supply.

Let $f(z)$ represent the frequency distribution of z over the entire field. The quantity $f(z)\,\mathrm{d}z$ represents the fraction of the field with a native soil-nutrient level between z and $z + \mathrm{d}z$. The mean indigenous nutrient supply is then $\bar{z}$, given by

$$\bar{z} = E_z\{z\} = \int_a^b z f(z)\,\mathrm{d}z\ , \qquad (1)$$

where a and b are the minimum and maximum values of z in the field.

To evaluate the logistics and economics of site-specific management the actual spatial structure of indigenous nutrient supply must be considered rather than merely its frequency distribution as given above. If we restrict our attention, however, to the effect of spatial variability on the mean yield of the whole field when fertilizer is applied uniformly, then only its frequency distribution needs to be considered.

We make the assumption that all other soil properties and crop management are homogeneous within the field except the indigenous nutrient supply. Let r represent the fertilizer (kg nutrient ha^{-1}) applied to the field. Let $Y(z, r)$ represent the crop yield (kg ha^{-1}) at any location in the field where the indigenous nutrient supply is z and the applied fertilizer is r. For a given input of r to a fraction $f(z)\,\mathrm{d}z$ of the field with indigenous supply between z and $z + \mathrm{d}z$, the yield contribution from that portion of the field is $Y(z, r)f(z)\,\mathrm{d}z$. Thus, the average yield over the whole field is the expected value over z of $Y(z, r)$. Denoting this expected value by E_z gives

$$E_z\{Y(z, r)\} = \int_a^b Y(z, r)f(z)\,\mathrm{d}z\ . \qquad (2)$$

and the total yield is obtained by multiplying this quantity by the fixed area of the field.

A significant portion of the applied nutrient, however, is often made unavailable to the crop by immobilization or fixation, or losses by leaching and other pathways. If the function $u(r)$ quantifies the relationship between the increase in plant-available nutrient supply and the amount of applied nutrient, then the yield from a heterogeneous field is

$$E_z\{Y(z, r)\} = \int_a^b Y(z, u(r))f(z)\,\mathrm{d}z\ . \qquad (3)$$

The relationship $u(r)$ can be derived from field experiments concerning FUE, but it must be valid over the range in z that is integrated in equation (3).

Cotton response to soil and fertilizer potassium

On irrigated soils derived from granitic alluvium in the San Joaquin Valley of California, yields of cotton (*Gossypium hirsutum* L.) are often severely limited by late-season K deficiency. Due to the large K fixation capacity of these vermiculitic soils [15], fertilizer-K exceeding

1400 kg ha^{-1} is sometimes required to achieve maximum seed-cotton yield [4]. The indigenous K supply, z, is most accurately reflected by a soil-test index that measures solution-phase K$^+$ concentration (SPK, in mg K L^{-1}), and the increase in SPK from fertilizer addition can be predicted from an adsorption isotherm [5]. Seed-cotton yield follows a Mitscherlich response in relation to SPK [3]. Data from a survey study that included 38 observations from farmers' fields [5] provide the following response function for seed-cotton yield Y (kg ha^{-1}) in relation to SPK of the surface 0 to 0.2 m layer (fig. 1a)

$$Y = 4940(1 - 0.835\,e^{-0.535\mathrm{SPK}})\,. \qquad (4)$$

Based on data from the adsorption isotherm (Fig. 1b), the relationship between SPK and the fertilizer input r is

$$\mathrm{SPK} = e^{\ln z + 0.0013r} = z\,e^{0.0013r}\,, \qquad (5)$$

assuming that the applied K is uniformly mixed in a surface soil mass of 2×10^6 kg ha^{-1}.

To run the model, a mean indigeneous K supply of $\bar{z} = 1.2$ mg K L^{-1} was specified. In a homogeneous field, this SPK value predicts a yield that is 56% of the maximum by equation (4). Yield was evaluated in hypothetical fields with heterogeneity in indigenous SPK specified to range from 0.5 to 1.9 mg K L^{-1}. This range was selected because such variation in SPK occurs in K-deficient fields of the San Joaquin Valley [4]. The function $u(r)$ is the increase in SPK from a fertilizer application of r, and is equal to $z(e^{0.0013r} - 1)$. Seed-cotton yield at any point in a heterogeneous field is estimated by combining equations (4) and (5) as

$$Y(z, u(r)) = 4940(1 - 0.835\exp(-0.535\,e^{\ln z + 0.0013r})) \qquad (6)$$

Effects of heterogeneity can then be evaluated by equation (3).

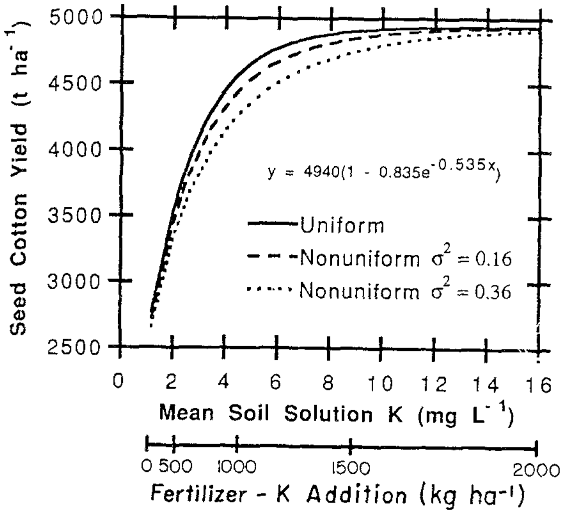

Fig. 1a. Seed cotton yield in relation to mean soil solution K$^+$ concentration of three fields that differ in uniformity of native soil-K levels. In all three fields, mean soil solution K$^+$ concentration is 1.2 mg K L^{-1} without added fertilizer-K (fields A, C, and D in Table 1), and the K input requirement from a uniform rate of fertilizer-K addition is shown on the lower abscissa. Data modifed from [3, 5].

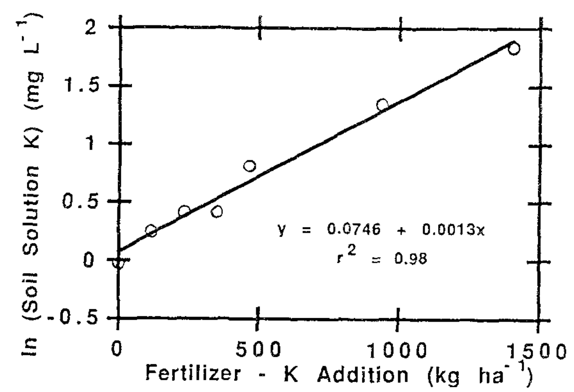

Fig. 1b. Relationship between the natural logarithm of soil solution K$^+$ concentration and the rate of fertilizer-K addition to a vermiculitic surface soil. Fertilizer-K addition levels are based on the assumption that applied K is uniformly incorporated in the 0 to 15 cm surface layer containing 2×10^6 kg soil ha^{-1}. Data modified from [5].

Crop response to soil and fertilizer nitrogen

Grain yield response of irrigated 'Anza' wheat (*Triticum aestivum* L.) to preplant N rates of 0, 60, 120, 180, and 240 kg N ha^{-1} was determined at three locations in California [8]. The response was similar across sites and followed a Mitscherlich function. Yield was closely related to the total accumulation of N in aboveground biomass measured at maturity (TPN, kg N ha^{-1}) as follows (Fig. 2a):

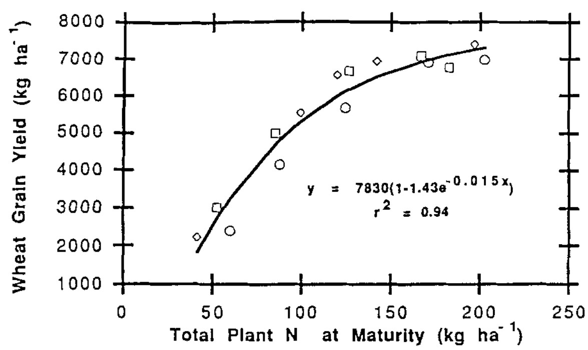

Fig. 2a. Grain yield of 'Anza' wheat in relation to total N accumulation in aboveground biomass at maturity from experiments conducted at three locations in California that are distinguished by separate symbols in the figure. Differences in total plant N reflect preplant N-rate treatments ranging from 0 to 240 kg N ha^{-1} at each location, and the yield response was similar across sites. Data from [8].

$$Y = 7830(1 - 1.43\,\mathrm{e}^{-0.015\mathrm{TPN}}). \tag{7}$$

The effect of fertilizer N (denoted r) on TPN was described by the regression of TPN on N rate (Fig. 2b) as

$$\mathrm{TPN} = 52.0 + 0.63r - 1.5(10^{-4})r^2 \tag{8}$$

so that the effective N uptake from fertilizer, $u(r)$, can be estimated at a specified N rate by the linear and quadratic terms of this relationship. The indigenous nutrient supply z is the crop N derived from soil resources. Thus, yield at any point in a heterogeneous field with a specified fertilizer input can be estimated by combining equations (7) and (8) to give

$$Y(z, u(r)) = 7830(1 - 1.43\,\mathrm{e}^{-0.015(z+0.62r-1.5(10^{-4})r^2)}) \tag{9}$$

This can be evaluated over variation in z by equation (3).

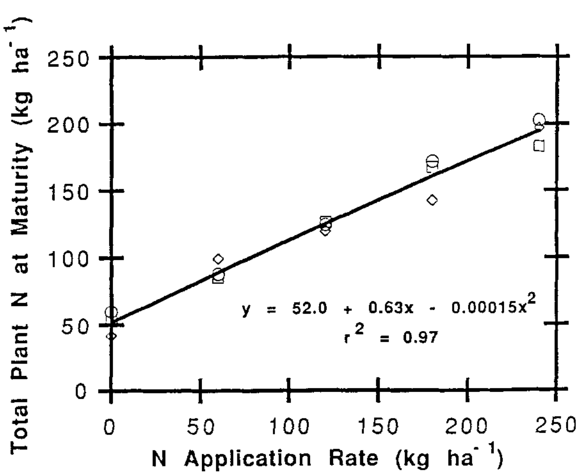

Fig. 2b. Apparent fertilizer-N uptake efficiency of 'Anza' wheat at three locations based on regression of total plant N accumulation in aboveground biomass on the rate of preplant N addition. Data from [8].

Response of irrigated lowland rice (*Oryza sativa* L.) was evaluated in a similar fashion. Three separate experiments with the same design were conducted in large commercial production fields at the Jari Rice Project in the state of Para, Brazil (1°S latitude, 52° longitude) located along the floodplain of the Amazon River. Except for the imposed N-level treatments, all field operations in the experimental area were identical to management in the surrounding commercial fields. In each experiment, N-rate treatments of 0, 30, 60, 90, 120, 150, and 180 kg N ha^{-1} were arranged in a randomized complete block design with three replicates. Nitrogen was applied in equal splits as urea at early tillering and at panicle initiation. At physiological maturity, all plants were cut at the soil surface, grain threshed, and grain and straw plus chaff oven-dried for yield determination and total N analysis.

Rice grain yield response to fertilizer-N best fitted a quadratic function. Reduced yield at higher fertilizer-N levels resulted from lodging during the grain filling period when intense rains often occurred. Yield (kg ha^{-1}) was closely related to total aboveground N accumulation at maturity (TPN, kg N ha^{-1}) by the quadratic formula

$$Y = -1261 + 127\mathrm{TPN} - 0.58\mathrm{TPN}^2. \tag{10}$$

This response was similar for the three production fields in which the experiment was conducted although the soil-N supply varied from 35 to 55 kg N ha^{-1} as reflected by the TPN of treatments without fertilizer-N addition (Fig. 3a). The influence of applied N on TPN of rice was represented by the quadratic relationship (Fig. 3b)

$$\mathrm{TPN} = 49.4 + 0.50r - 4.2(10^{-4})r^2\,, \qquad (11)$$

and thus the effective N uptake from fertilizer is estimated by $u(r) = 0.50r - 4.2(10^{-4})r^2$. Evaluation of yield over variability in z is accomplished by combining equations (10) and (11) to give

$$Y(z, u(r)) = -1261 + 127(z + u(r)) - 0.58(z + u(r))^2 \qquad (12)$$

which is integrated over the frequency distribution of z as in equation (3).

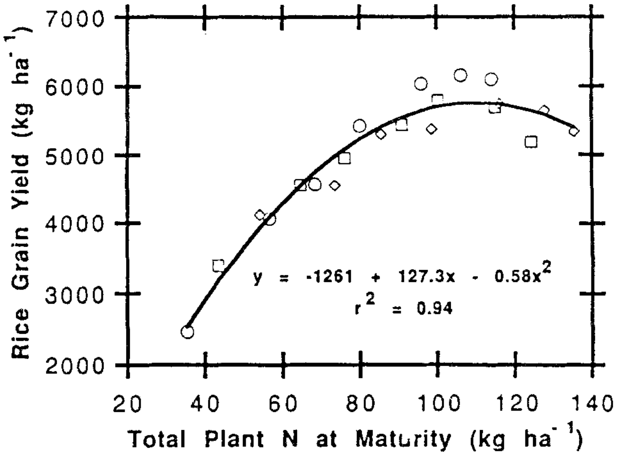

Fig. 3a. Grain yield of irrigated rice in relation to total N accumulation in aboveground biomass at maturity. Differences in total plant N reflect N-rate treatments ranging from 0 to 180 kg N ha^{-1}, and symbols represent treatment means from three separate experiments conducted in different production fields located at the same farm.

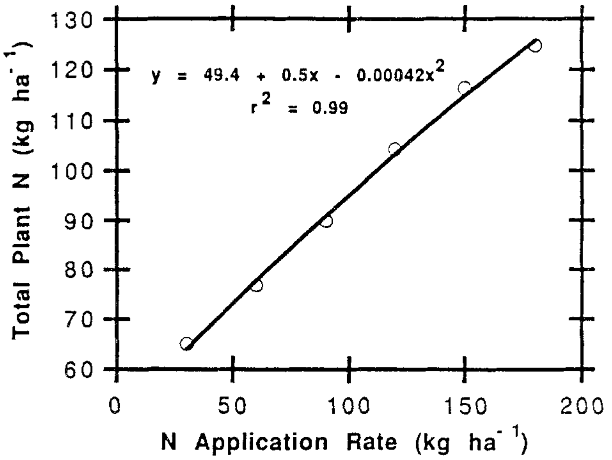

Fig. 3b. Apparent fertilizer-N uptake efficiency of irrigated rice based on regression of total plant N accumulation in aboveground biomass at maturity on the rate of fertilizer-N addition. Values shown are pooled means from the three experiments shown in Figure 3a. Pooling of data was justified because the slopes were comparable for the three experiments although the Y-intercepts differed significantly.

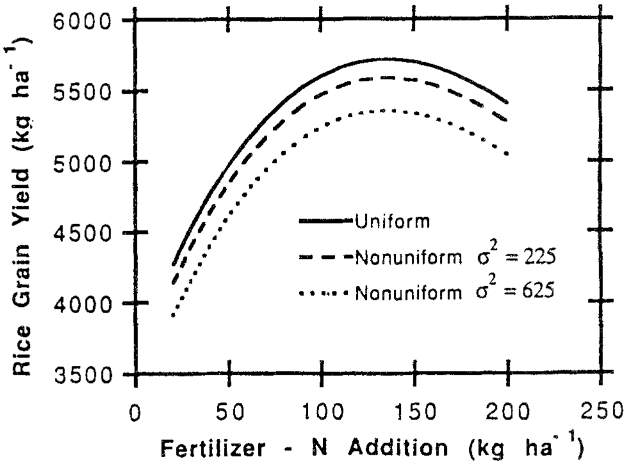

Fig. 3c. Model output predicting grain yield of irrigated rice in relation to fertilizer-N addition that is uniformly applied to fields which differ in the degree of uniformity in native soil-N supply (fields A, C, and D in Table 3).

Results

Evaluation of the Model

Assume that the price per kg of yield is independent of the crop yield, Y, and denote this price by p. Let the cost per ha of fertilizer application be denoted $c(r)$, where r is the fertilizer level. If fertilizer is to be applied uniformly, then the problem of choosing the level of fertilizer that maximizes economic return is, in the model, equivalent to choosing the value of r that maximizes the quantity

$$J(r) = \int_a^b pY(z, u(r))f(z)\,\mathrm{d}z - c(r)\,. \qquad (13)$$

To study the effects of spatial heterogeneity in indigenous nutrient supply on the optimal level of fertilizer when a nutrient is applied uniformly, the ratio of nutrient cost to crop price is important. For this paper, however, the object is simply to study the effect of heterogeneity on economic yield and output/input ratios for a given level of applied nutrient. That is, for a fertilizer input r, the quantity $J(r)$ above is the economic yield from a heterogeneous field where fertilizer is applied uniformly. By contrast, the economic yield obtained from a homogeneous field with the same mean indigenous nutrient supply is given by

$$J_0(r) = pY(\bar{z}, u(r)) - c(r) . \tag{14}$$

The difference between $J_0(r)$ and $J(r)$ gives an indication of the loss in economic yield due to soil heterogeneity. By denoting the yield from a heterogeneous field as the expected value in equation (3), the difference between $J_0(r)$ and $J(r)$ is:

$$J_0(r) - J(r) = p[Y(\bar{z}, u(r)) - E_z\{Y(z, u(r))\}] . \tag{15}$$

At any fixed price for the produced commodity, the decrease in economic return that results from soil heterogeneity depends solely on the difference in yield between the homogeneous and heterogeneous field, both receiving an equivalent input of fertilizer which is uniformly applied. We will therefore focus our attention on the effects of spatial variability, quantified by the distribution $f(z)$ of indigenous nutrient supply, on the difference between $Y(\bar{z}, u(r))$ and $E_z\{Y(z, u(r))\}$.

Let $D(r)$ denote this difference, that is

$$D(r) = Y(\bar{Z}, u(r)) - E_z\{Y(z, u(r))\} , \tag{16}$$

It is immediately evident, from the basic integral formulas for the two quantities in this difference, that spatial variability only affects yield to the extent that the yield response is *nonlinear* in relation to z. Likewise, if the total nutrient supply from indigenous resources and fertilizer input are so low that the crop response to the applied nutrient is mostly linear, then spatial heterogeneity would have little effect on yield.

We can determine how the distribution $f(z)$ of the native soil-nutrient level affects the overall yield difference $D_r\{Y(z, u(r))\}$ for a given application rate r by expanding the yield function $Y(z, u(r))$ in a Taylor series about the mean native soil nutrient level z. This expansion yields

$$\begin{aligned} Y(z, u(r)) = Y(\bar{z}, u(r)) & \\ + Y'(\bar{z}, u(r))(z - \bar{z}) & \\ + \frac{1}{2} Y''(\bar{z}, u(r))(z - \bar{z})^2 & \\ + \frac{1}{6} Y'''(\bar{z}, u(r))(z - \bar{z})^3 + \dots & \end{aligned} \tag{17}$$

where Y', Y'', and Y''' are the first, second, and third derivatives of the yield function with respect to z and the ellipsis denotes terms of degree four or higher in $F(z - \bar{z})$. Equation (17) may be substituted into equation (16) for the difference $D(r)$ between the expected yield of a homogeneous field and a heterogeneous field, both with the same $\bar{z}$. Making this substitution, performing the integrations, and doing some algebra yields the following equation:

$$\begin{aligned} D(r) = -\Bigg[\frac{1}{2} & Y''(\bar{z}, u(r))\mathrm{var}\{z\} \\ & + \frac{1}{6} Y'''(\bar{z}, u(r))\mu_3\{z\}\Bigg] + \dots \end{aligned} \tag{18}$$

where var$\{z\}$ is the variance of the distribution $f(z)$ *and* $\mu_3\{z\}$ is the third moment of the distribution $f(z)$, which is a measure of its skewness. Thus, if the distribution $f(z)$ is symmetrical about the mean so that skewness is zero, then the yield difference $D(r)$ between a homogeneous and heterogeneous field that both receive a uniformly applied nutrient is approximately proportional to the variance in the distribution $f(z)$ of the indigenous nutrient supply. If the distribution $f(z)$ is skewed, then the difference $D(r)$ is either augmented or reduced depending on the signs of the skewness and the second and third derivatives. Since the yield response functions Y are analytic, they possess uniformly convergent Taylor series so that effects of the higher derivatives will be relatively small.

Application of the model

The equations given in the preceding subsection, together with equation 16 for the differential yield $D(r)$ and the crop response function to total nutrient supply from soil and fertilizer, specify the behavior of the model for a given indigenous nutrient distribution $f(z)$ and fertilizer input r. Specifying $f(z)$, $u(r)$, and $Y(z, u(r))$ completely specifies the model. To perform an analysis of the model, it was judged desirable to pick simple, schematic forms for the distribution $f(z)$, and then for each function $f(z)$, to observe the behavior of the model over a range of values of r. The Taylor expansion formula for $D(r)$ given in the preceding section shows that $D(r)$ depends on the distribution $f(z)$ through its second and higher moments. For this reason, the form chosen for $f(z)$ is a simple two point distribution in which all of the 'probability mass' is concentrated at two indigenous nutrient supply values, one above the mean and one below the mean. The variance and skewness of this distribution are determined by the spread and relative mass concentration of the two points as illustrated in Figure 4.

This simple form for the function $f(z)$ clearly does not represent the form that such a distribution would take in a real field. It does, however, provide a simplified representation of the primary factors, variance and skewness, controlling the effect of this distribution on yield.

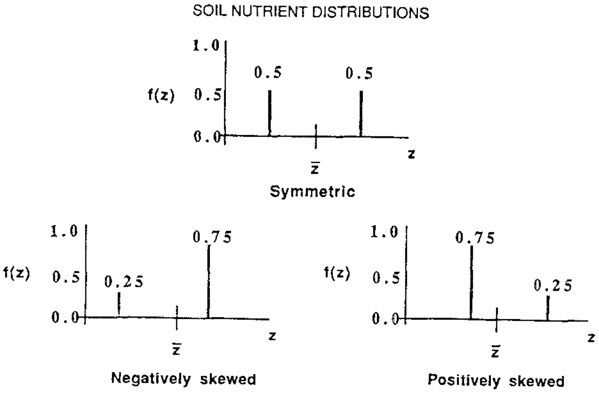

Fig. 4. Schematic diagram representing the two-point probability distribution mass fequency in native soil-nutrient levels used to evaluate crop response to applied fertilizer-nutrient in nonuniform fields.

Although the model is valid for evaluating more complex spatial distributions of soil heterogeneity, this simple two-point distribution of z is more useful for a comparative analysis of soil heterogeneity effects on yield and FUE when a nutrient is applied uniformly over the entire field, or precisely applied at different input levels to achieve an equivalent yield in each location within the field.

Soil heterogeneity and cotton response to potassium

When the model is used to evaluate the K response of cotton on a K-fixing soil, heterogeneity in indigenous K supply results in decreased yield whenever soil K availability in any portion of the field is below the threshold required for near-maximum yield. With a mean indigenous SPK of 1.2 mg K L^{-1} specified for the whole field, the yield penalty from heterogeneity increases with greater variance in indigenous nutrient supply when K fertilizer is applied uniformly (Fig. 1a).

In accord with the Taylor series expansion (equation 18), the magnitude of the second moment, or variance, in indigenous K supply has the greatest impact on seed cotton yield, and therefore also on the fertilizer-K requirement. This is illustrated by calculating the fertilizer-K requirement for hypothetical fields with the same mean native soil-K level but with a range in variance of indigenous K and targeted yield levels (Table 1). For example, when fertilizer-K is applied at a uniform rate to achieve a yield that is 85% of the maximum, the K input requirement increases by 3%, 11%, and 27% due to increased variance in indigenous K supply as the CV increases from 17%, to 33%, to 50% of the mean, respectively.

The spatial structure of heterogeneity in indigenous K supply influences the K requirement because the cotton yield response to K supply is a nonlinear function (equation 4), with the skewness of the distribution $f(z)$ represented by the term μ_3 in the Taylor expansion series of equation 18. With a negatively skewed distribution such that a greater proportion of the field area has an indigenous K supply greater than the mean (Fig. 4), the K input requirement is further increased above the requirement for a field with

Table 1. Influence of field variance and skewness in native soil K supply and method of K application on the K input requirement and fertilizer-K utilization efficiency at three specified levels of relative seed cotton yield[†]

Field	Soil K uniformity[††]			Fertilizer Application method[†††]	K Fertilizer requirement			ΔYield/K input ratio		
	mean	variance (CV%)	skewness		--relative yield level-- 75%	85%	95%	--relative yield level-- 75%	85%	95%
	------------- $\mathrm{mg\,K\,L^{-1}}$ -------------				------- $\mathrm{kg\,K\,ha^{-1}}$ -------			------- $\mathrm{kg\,kg^{-1}}$ ------		
A	1.20	0	0	uniform	484	754	1134	1.93	1.90	1.70
B	1.20	0.04 (17)	0	uniform	497	773	1164	1.91	1.87	1.65
				precise	495	765	1145	1.92	1.89	1.69
C	1.20	0.16 (33)	0	uniform	539	835	1264	1.83	1.77	1.56
				precise	529	800	1179	1.86	1.85	1.67
D	1.20	0.36 (50)	0	uniform	624	961	1454	1.68	1.61	1.40
				precise	594	865	1245	1.77	1.78	1.64
E	1.20	0.16 (33)	−1.16	uniform	551	862	1354	1.80	1.72	1.46
				precise	548	818	1198	1.81	1.81	1.65
F	1.20	0.16 (33)	+1.16	uniform	531	818	1224	1.85	1.80	1.61
				precise	519	790	1170	1.89	1.87	1.68

[†] Maximum yield represents a 100% relative yield, and is estimated by the asymptote of equation 4.

[††] Soil K supply is considered to be the solution-phase K^+ concentration in the 0–20 cm topsoil.

[†††] Fertilizer-K applied and incorporated uniformly across the field (uniform), or precisely adjusted (precise) to achieve the specified relative yield level in each portion of the heterogeneous field.

normally distributed variation in soil K (field E versus field C, Table 1). Conversely, positive skewness attenuates the negative impact of the pure variance effect. This is due to the opposite signs of the second and third derivatives of the Mitscherlich yield response.

As the fertilizer-K requirement increases due to greater heterogeneity in indigenous K supply, there is a decline in FUE as reflected in the Δyield/ΔK input ratio (Table 1). Graphic depiction of the relationship between the degree of variation in indigenous K supply and the relative FUE, where 1.0 represents the Δyield/ΔK input ratio from a uniform application of K fertilizer to a homogeneous field, is presented in Figure 5a.

The ability to apply K at variable rates that precisely match the input requirement of each location is a heterogeneous field substantially reduces the K input requirement of the whole field compared to a uniform K application, but precision application does not completely overcome the negative effects of heterogeneity (Table 1). This penalty in fertilizer-K requirement and FUE (Fig. 5a), even when K inputs are precision-applied, results from the strong K fixation character of these vermiculitic soils (Fig. 1b). Relatively more of the applied K is fixed in unavailable forms in areas of the field with indigenous K supply below the mean, while the benefit of less K fixation in areas with indigenous K supply above the mean is reduced by the diminishing return nature of the Mitscherlich yield response to K supply. The economic cost that results from uniform fertilizer-K application to a heterogeneous field is therefore overestimated when this cost is based on the difference in K input requirements of the homogeneous and heterogeneous fields. The appropriate comparison is between the nutrient input requirements of uniform versus precision application methods to a heterogeneous field.

Soil heterogeneity and wheat response to nitrogen

Like the cotton response to K, wheat response to N supply also followed a Mitscherlich function with a defined yield plateau (Fig. 2a). The model predicts that the effects of variance and skewness in indigenous N supply on the fertilizer-N requirement (Table 2) follow similar trends as in the cotton example. The magnitude of the effects of heterogeneity on the wheat response, however, was much smaller at comparable levels of variance and skewness. For wheat, the mean soil-N supply was specified at 75 $\mathrm{kg\,N\,ha^{-1}}$ which produces a grain yield in a uniform field that is 54% of maximum (equation 7), versus a 56%

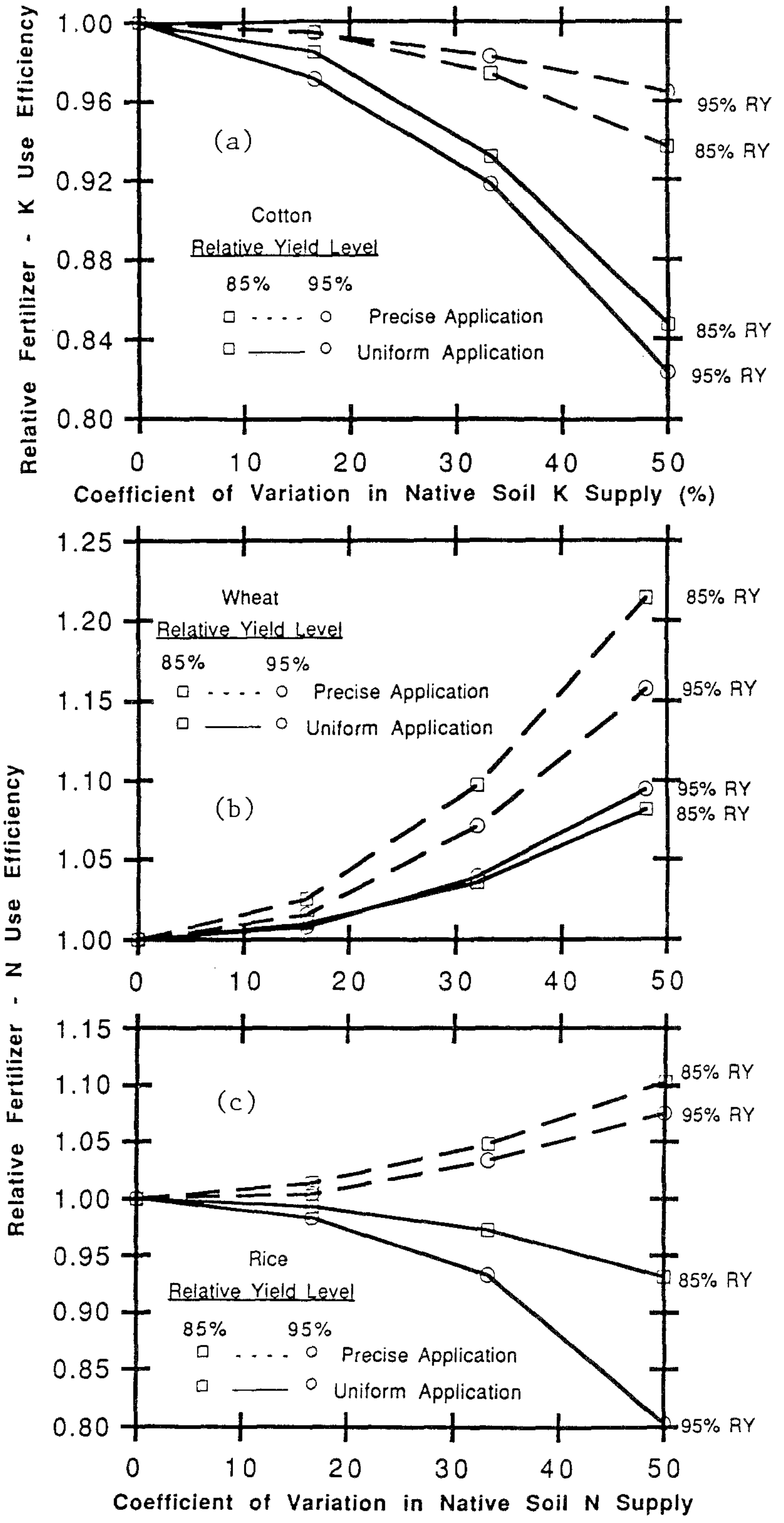

Fig. 5(a–c). Model output predicting the effect of heterogeneity in native soil-nutrient supply on the relative fertilizer-nutrient utilization efficiency (RFUE), where 1.0 represents the Δyield/Δnutrient input ratio of a completely uniform field at the indicated relative yield level. RFUE values are based on Δyield/Δnutrient input ratios presented in Tables 1, 2, and 3 for cotton (fig. 5a), wheat (Fig. 5b), and rice (Fig. 5c), respectively.

Table 2. Influence of field variance and skewness in native soil N supply and method of N application on the N input requirement and fertilizer-N utilization efficiency at three specified levels of relative wheat grain yield[†]

Field	Soil N uniformity[††]			Fertilizer Application method[†††]	N Fertilizer requirement			ΔYield/N input ratio		
	mean	variance (CV%)	skewness		--relative yield level--			--relative yield level--		
					75%	85%	95%	75%	85%	95%
	------kg N ha^{-1} --------				---kg-N ha^{-1}----			----- kg kg^{-1}---		
A	75	0	0	uniform	67	125	255	24.8	19.6	12.7
B	75	144 (16)	0	uniform	69	127	257	25.0	19.8	12.8
				precise	68	125	255	25.6	20.1	12.9
C	75	576 (32)	0	uniform	74	132	263	25.6	20.3	13.2
				precise	68	126	256	28.1	21.5	13.6
D	75	1296 (48)	0	uniform	83	141	272	26.7	21.2	13.9
				precise	68	126	256	32.4	23.8	14.7
E	75	576 (32)	−1.16	uniform	76	133	264	25.7	20.5	13.3
				precise	68	126	256	28.7	21.7	13.7
F	75	576 (32)	+1.16	uniform	74	131	262	25.5	20.2	13.1
				precise	68	126	256	27.7	21.2	13.5

[†] Maximum yield represents a 100% relative yield that is estimated by the asymptote of equation 7.

[††] Native soil N supply is considered to be the total N accumulation in aboveground wheat biomass at maturity from soil without fertilizer-N addition.

[†††] Fertilizer-N applied and incorporated uniformly across the field (uniform), or precisely adjusted (precise) to achieve the specified relative yield level in each portion of the heterogeneous field.

relative yield in a uniform field without fertilizer-K addition for the cotton example in Table 1 and Figure 1a.

Less sensitivity of the wheat N response to soil heterogeneity reflects the relatively high fertilizer-N uptake efficiency which was mostly linear over a wide range of N input levels (Fig. 2b). The strong linear character of this response also results in greater FUE as the degree of heterogeneity in indigenous N supply increases (Table 2 and Fig. 5b) because there is a greater Δyield/ΔN input ratio in areas where indigenous N is below the mean of the whole field, and this increase more than compensates for the smaller Δyield/ΔN input ratio in areas with indigenous N supply above the whole-field mean. Thus, FUE increases as heterogeneity increases in the wheat example. This trend is opposite the prediction from the cotton K-response (Fig. 5a) where soil fixation of applied K makes the net increase in K supply extremely dynamic with respect to the initial indigenous soil K value (fig. 1b).

Also in contrast to the K response of cotton, precision application of fertilizer-N to wheat nearly eliminates the effect of heterogeneity on the fertilizer-N input requirement (Table 2). The greater efficacy of precision N application in the wheat system is again a result of the mostly linear relationship between crop N uptake and N input levels, and the assumption that this relationship holds over the range of indigenous N supply values specified in the evaluation.

Soil heterogeneity and rice response to nitrogen

The parabolic response of rice yield to N supply (Fig. 3a) is an empirical relationship that accommodates the observed yield reductions due to lodging when the N supply exceeds an optimal level. Because the quadratic yield response does not have a third derivative, the predicted yield difference between homogeneous and heterogeneous fields at any level of applied N is a constant value that depends entirely on the magnitude of the variance in indigenous N supply (Fig. 3c).

Effects of heterogeneity on the N response of rice were evaluated over a range in variance of indigenous supply (Table 3) that is comparable to the wheat and cotton examples. Likewise, the mean indigenous N supply of the whole-field was specified at 42 kg N ha^{-1} which gives a relative rice yield that is 54% of maximum in a homogeneous field without applied N, similar to the relative yields of unfertilized wheat and cotton in the previous examples.

Table 3. Influence of field variance in native soil N supply and method of N application on the N input requirement and fertilizer-N utilization efficiency at three specified levels of relative rice grain yield[†]

Field	Soil N uniformity[††]		Fertilizer Application method[†††]	N Fertilizer requirement			Δ Yield/N input ratio		
				---relative yield level---			---relative yield level---		
	mean	variance (CV%)		75%	85%	95%	75%	85%	95%
	---kg N ha^{-1}---			------kg N ha^{-1}-----			------kg kg^{-1}------		
A	42	0	uniform	37.5	62.0	99.6	32.9	29.1	23.9
B	42	49 (17)	uniform	38.6	63.4	102.4	32.7	28.9	23.5
			precise	37.7	62.2	100.0	33.5	29.5	24.0
C	42	196 (33)	uniform	41.8	67.9	111.8	32.2	28.3	22.3
			precise	38.3	62.9	100.9	35.2	30.5	24.7
D	42	441 (50)	uniform	47.5	76.1	137.1	31.3	27.1	19.2
			precise	39.0	64.1	102.3	38.2	32.1	25.7

[†] Maximum yield represents a 100% relative yield and is estimated by the first derivative of the quadratic yield response to N supply when $\Delta\text{Yield}/\Delta\text{TPN} = 0$ in equation 10.

[††] Native soil N supply is considered to be the total N accumulation in aboveground rice biomass at maturity from soil without fertilizer-N addition.

[†††] Fertilizer-N applied and incorporated uniformly across the field (uniform), or precisely adjusted (precise) to achieve the specified relative yield level in each portion of the heterogeneous field.

Increasing the magnitude of variance in indigenous N supply results in a substantial increase in the N input requirement when N is applied uniformly to the whole field (Table 3). Precision application overcomes most of the increase in fertilizer-N requirement due to soil heterogeneity, but precision application is somewhat less effective than in the wheat example. This difference reflects the lower nitrogen FUE of rice as reflected by a smaller linear coefficient and a larger quadratic coefficient in the relationship between N accumulation and N input level (fig. 3b versus Fig. 2b).

Reduced fertilizer-N uptake efficiency by rice and a parabolic yield response to N supply causes opposing trends in FUE for uniform versus precision application to heterogeneous fields (Table 3 and Fig. 5c). As the variance in indigenous N supply increases, FUE decreases relative to a homogeneous field when N inputs are applied uniformly to the entire field. This trend is reversed with precision application. These opposing trends increase the relative difference in FUE for precision versus uniform N application methods to rice considerably more than in the wheat case study (Fig. 5c versus Fig. 5b).

Discussion

To reduce the costs that result from greater nutrient input requirements of heterogeneous fields, three general strategies could be employed: (1) fields could be subdivided to reduce soil variability within subunits, each of which is managed uniformly; (2) technologies could be introduced that increase the overall efficiency of input utilization while adhering to uniform management of the whole field; (3) input levels could be applied at variables rates, precisely calibrated to optimize productivity and minimize input requirements at each location in the field. Each of these strategies is likely to require investment in capital improvements, specialized equipment, and/or greater labor costs, and thus a method is needed to compare the cost-effectiveness of these options.

The basis for these comparisons is the estimation of yield from a heterogeneous field that receives an input applied at a uniform rate. For a fertilizer-nutrient, this yield estimate requires specifying (1) the yield response function to the total nutrient supply including the contribution from soil and fertilizer, (2) the relationship between the quantity of applied nutrient and the net increase in plant-available nutrient supply, and (3) the frequency distribution of indigenous nutrient supply in the field.

The nutrient supply parameter can be directly quantified as crop nutrient uptake, as in the wheat and rice N-response examples, or indirectly as a soil-test index that is well correlated with crop yield response. In the cotton example, a soil-test K index was used as the indicator of nutrient supply, similar to the approach taken by Ndiaye and Yost [14] in their evaluation of nonuniformity in fertilizer-K application and effects on crop response.

Whether based on a direct or indirect measure of nutrient supply, it is assumed in the model that the relationship between the net increase in plant-available nutrient supply and nutrient input level is valid at all locations within the field. Another assumption is that the yield potential and the yield response function to nutrient supply is equivalent in all areas of a heterogeneous field. Both assumptions imply that indigenous nutrient supply is the only difference associated with soil heterogeneity, which is of course, a gross simplification. Where soil heterogeneity results from substantial differences in soil texture, profile depth, or mineralogy, the above assumptions are not likely to be valid. In the wheat- and rice-N response examples, however, the slope of the relationship between plant N uptake and N input level was found to be similar across locations (fig. 2b) and fields (Fig. 3b) indicating that use of a single function may be justified in some cases.

Differences in the yield response function to nutrient supply and in the yield potential due to soil properties other than indigenous nutrient supply could be accommodated in the model if the relationship between these auxiliary factors and indigenous nutrient supply can be defined with reasonable precision. For example, differences in soil texture, organic matter content, and indigenous soil N supply are often closely related [2, 12]. In rainfed systems, soil texture and organic matter content largely govern water storage capacity and may therefore influence yield potential. Knowledge of the quantitative relationship among indigenous N supply, soil water-holding capacity, and yield potential could be incorporated in the model to adjust the yield response function with respect to these interactions.

Despite potential limitations of the assumptions in the model, evaluation of three case studies from irrigated systems emphasizes the dynamic interactions among variables governing crop response to nutrient inputs that are uniformly applied to heterogeneous fields. The magnitude of the predicted effect of heterogeneity on the nutrient input requirement of the whole field depends on interactions among (1) the mathematical form of the response function $Y(z, u(r))$, (2) the degree and spatial structure of the heterogeneity in indigenous nutrient supply as quantified by its variance, skewness, and possibly higher order moments of the response function, (3) the efficacy of fertilizer-nutrient addition as determined by $u(r)$ and the quantity of applied nutrient, and (4) the targeted yield level.

In each of the case studies, the model predicts that the nutrient input requirements of the whole field increases with greater heterogeneity in soil nutrient supply, and this increased requirement is greatest at higher targeted yield levels and when fertilizer uptake efficiency is low. Indeed, technologies that increase nutrient uptake efficiency, such as banding or split applications, provide one option for reducing the adverse effects of soil heterogeneity on the nutrient input requirement. For example, increasing the linear coefficient of the relationship between rice N uptake and fertilizer-N input by 15%, from 0.50 to 0.575 (Fig. 3b), reduces the difference in N input requirement between homogeneous and heterogeneous fields by 20% when N is applied uniformly and the CV of indigenous N supply is 33 to 50%.

The cost benefit from precision application of fertilizer, however, will depend on the difference in the FUE of precise versus uniform application methods. This difference determines the economic return expected from precision application with specified unit prices for the commodity produced and the fertilizer-nutrient. Prediction of output/input ratios by the model for the three case studies indicates complex interactions among the factors governing crop response to applied nutrients in a heterogeneous field. In one case, the model predicts that the relative benefit of precise N application to wheat is greatest at lower relative yield levels although the FUE increases as variance in indigenous nutrient supply increases regardless of yield target (Fig. 5b, Table 2). By contrast, relative differences in FUE of the rice-N response are greatest at higher relative yield levels, and FUE increases with increasing variance in indigenous N supply *only* when fertilizer-N is precision-applied (Fig. 5c, Table 3). In the cotton example where K fixation by the vermiculitic soil makes FUE extremely low, the output/input ratio declines with increasing soil heterogeneity regardless of K application method although precision application greatly reduces the effects of heterogeneity (Fig. 5a, Table 1).

Justification for precision fertilizer-N application to match site-specific requirements may not depend on economic considerations alone, however, when the potential for nitrate pollution of groundwater is a concern. Where the degree of soil heterogeneity in nutrient supply is large, uniform N application to achieve high relative yields may lead to excessive addition rates to areas of the field high in indigenous N supply, including residual N from a previous crop. In these areas, the potential for nitrate leaching could be much greater than expectations based on the mean soil-N supply of the whole field. The model framework we present for assessing crop response to fertilizer in a heterogeneous field could be modified to evaluate the effects of soil heterogeneity on nutrient leaching loses by the addition of submodels that predict the fate of applied N not utilized by the crop.

References

1. Bresler E and Laufer A (1988) Statistical inferences of soil properties and crop yields as spatial random functions. Soil Sci Soc of Am J 52: 1234–1244
2. Burke IC, Yonker CM, Parton WJ, Cole CV, Flach K and Shimel DS (1989) Texture, climate, and cultivation effects on soil organic matter content in U.S. grassland soils. Soil Sci Soc Am J 53: 800–805
3. Cassman KG, Kerby TA, Roberts BA, Bryant DC and Brouder SM (1989) Differential response of two cotton cultivars to fertilizer and soil potassium. Agron J 81: 870–876
4. Cassman KG, Roberts BA, Kerby TA, Bryant DC and Higashi SL (1989) Soil potassium balance and cumulative cotton response to annual potassium additions on a vermiculitic soil. Soil Sci Soc Am J 53: 805–812.
5. Cassman KG, Bryant DC and Roberts BA (1990) Comparison of soil test methods for predicting cotton response to soil and fertilizer potassium on potassium fixing soils. Comm Soil Sci and Plant Anal 21: 1727–1743
6. Dinar A, Letey J and Knapp KC (1985) Economic evaluation of salinity, drainage, and nonuniformity of infiltrated irrigation water. Agric Water Management 10: 221–223
7. Harris JA (1920) Practical universality of field heterogeneity as a factor influencing plot yields. J Agric Res 19: 279–314
8. Hawe WP, Cassman KG, Fulton A and Jackson LF (1989) Evaluation of tissue nitrogen tests as a prognostic tool for irrigated wheat production. J Fert Issues 6(4): 89–99
9. Hughes G (1989) Spatial heterogeneity in yield-weed relationships for crop-loss assessment. Crop Res 29(2): 87–94
10. Kolasa, J and Rollo, CD (1991) The heterogenity of heterogenity: A glossary. pp 1–23, Ecological Heterogenity, J Kolasa and STA Pickett, eds, Springer Verlag, New York
11. Luellen WR (1985) Fine-tuned fertility: tomorrow's technology here today. Crops and Soils 38(2): 18–22
12. Meng TP, Taylor HM, Fryrear DW and Gomez JF (1987) Models to predict water retention in semiarid sandy soils. Soil Sci Soc Am J 51: 1563–1565
13. Mulla DJ (1988) Estimating spatial patterns in water content, matric suction, and hydraulic conductivity. Soil Sci Soc Am J 52: 1547–1553
14. Ndiaye JP and Yost RS (1989) Influence of fertilizer application nonuniformity on crop response. Soil Sci Soc of Am J 53: 1872–1878
15. Shaviv A, Moshin M, Pratt PF and Mattigod SV (1985) Potassium fixation characteristics of five southern California soils. Soil Sci Soc Am J 49: 1105–1109
16. Wichelns D and Oster JD (1991) Irrigation uniformity and cotton yields in the San Joaquin Valley. Calif Agric 45(1): 13–15